Multiply scattered light correlations in an expanded temporal range

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The multiple scattering of light from a Brownian particle suspension is considered for backscattering as well as transmission through a finite-thickness slab. A method for calculation of the radiative transfer propagator is developed permitting to extend significantly a range of time wherein the temporal correlation function can be found. Using the elaborated approach, numerical results are obtained in good agreement with experiment. A deviation of a correlation function initial slope from linear is shown to arise from a contribution of a finite number of scattering orders. The correlation functions for polarized and depolarized scattered light components are calculated inside and outside the backscattering cone. The transmittion correlation function is found for time intervals far exceeding a characteristic time of Brownian diffusion. A double-scattering term of the temporal correlation function is obtained that is valid for any time interval. [S1063-651X(97)02109-0]

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I. INTRODUCTION

Much attention has been paid recently to light intensity temporal correlations [1–12] in highly turbid media. Their study is an essential part of the general problem of coherent phenomena in multiple scattering (see reviews [12–15]). The temporal correlation functions of light scattered from a concentrated Brownian particle suspension are mostly studied [1,2,4,7]. They are usually considered for a time interval less than the characteristic time τ needed for a particle to diffuse a light wavelength distance. The correlation function is shown to decay with time t as const$-\sqrt{t/\tau}$ in this time interval in accordance with a theory [1,3] predicting within the diffusion approximation a linear dependence on $\sqrt{t/\tau}$. However, an attempt to describe the light correlation within the same approach in a wider temporal range studied experimentally turned to be unsuccessful, leading to a noticable discrepancy between theory and experiment [5].

Theoretical studies [1,3,7,8] were carried out mostly for a scalar field, leaving effects of light polarization beyond the scope of consideration. The vector nature of an electromagnetic field was taken into account in Ref. [16], describing the integral intensities of polarized and depolarized light components in coherent backscattering. The polarized component only was shown to exhibit a peculiar ''triangular'' dependence of the backscattering peak on the scattering angle, whereas the depolarized component was predicted to take a Lorenzian form, in good agreement with experiment. For a system with absolutely anisotropic fluctuations of permittivity, the coherent backscattering peak was shown [17] to vanish at all for depolarized components.

Considering the temporal correlation function far from the backscattering cone, initial slopes of the decay rate were calculated for polarized and depolarized components in Ref. [5]. Contrary to coherent backscattering, the decay of the depolarized component turns out to be steeper than that of the polarized one. Describing coherent effects in multiple scattering, the radiative transfer propagator is taken generally in its asymptotic form of r^{-1}. It corresponds to the small momentum transfer region in wave-vector space. Such an approximation may turn to be insufficient for quantitative purposes, since the momentum transfer should contribute up to a value of order $q\sim 1/l$, where l is the photon mean free path, or extinction length.

We consider light scattering from a Brownian particle suspension. The temporal correlation function is presented as a series in scattering orders. Accounting successively for an increasing number of scattering orders, we find a remarkable correspondence between the scattering order number under consideration and a degree of deviation from a linear behavior of the initial slope of the temporal correlation function observed experimentally for backscattering from slabs of different thicknesses. The relative weight of the lower scattering order contribution is shown to increase with time. Going beyond the first t/τ order approximation and summing the multiple-scattering series, we obtain numerical results that agree rather well with the known measurement data. Considering the electromagnetic field, the decay rate of the temporal correlation function is revealed to depend essentially on the light polarization as well as the scattering angle.

We also consider transmission through a slab of finite thickness, obtaining a closed expression for the temporal correlation function in a large-time limit. We calculate the double-scattering term for an arbitrary time interval as a function of slab thickness L.

The paper is organized as follows. In Sec. II a general method is outlined, deriving the temporal correlation function of multiply scattered radiation. In Sec. III the temporal correlation function is calculated for backscattering within the scalar field approach. In Sec. IV we take into account the vector nature of an electromagnetic field, considering the temporal correlation function in the neighborhood of the coherent backscattering peak. In Sec. V we consider the temporal asymptotics of the correlation function for radiation transmitted through a finite thickness slab, and calculate the double-scattering term for an arbitrary time value. Section VI is devoted to a general discussion of the results obtained.

II. MULTIPLE SCATTERING SERIES FOR CORRELATION FUNCTION

We consider a temporal correlation of the intensity defined as

$$C_I^{(\alpha\beta)}(t)=\langle \delta I_\alpha(0)\,\delta I_\beta(t)\rangle-\langle \delta I_\alpha(0)\rangle\langle \delta I_\beta(t)\rangle, \quad (2.1)$$

where $\delta I_\beta(t)$ is the scattered light intensity at moment t, lower indices determine the scattered light polarization, and brackets mean averaging over realizations. Presenting the intensity as a product of electric fields,

$$\delta I_\alpha(t)=\delta E_\alpha(t)\,\delta E_\alpha^*(t),$$

where $\delta E_\alpha(t)$ is the scattered electric field with polarization α at moment t, one obtains the intensity correlation function as an average of fourth order in field

$$C_I^{(\alpha\beta)}(t)=\langle \delta E_\alpha(0)\,\delta E_\alpha^*(0)\,\delta E_\beta(t)\,\delta E_\beta^*(t)\rangle-\langle|\delta E_\alpha(0)|^2\rangle \times\langle|\delta E_\beta(t)|^2\rangle. \quad (2.2)$$

One can present this correlation function within the Gaussian approximation as the pairwise correlation product

$$C_I^{(\alpha\beta)}(t)=|\langle \delta E_\alpha(0)\,\delta E_\beta^*(t)\rangle|^2. \quad (2.3)$$

Such an approximation for multiple scattering was used first by Shapiro [18].

Field $\mathbf{E}(\mathbf{r},t)$ is a solution of the wave equation for a random medium,

$$\text{curl}\quad\text{curl}\ \mathbf{E}(\mathbf{r},t)+\frac{1}{c^2}\frac{\partial^2\mathbf{E}(r,t)}{\partial t^2}=-\frac{4\pi}{c^2}\frac{\partial^2\mathbf{P}(\mathbf{r},t)}{\partial t^2}, \quad (2.4)$$

where two random vectors, polarization $\mathbf{P}(\mathbf{r},t)$ and field $\mathbf{E}(\mathbf{r},t)$, are connected by the relationship

$$\mathbf{P}(\mathbf{r},t)=\frac{\varepsilon(\mathbf{r},t)-1}{4\pi}\mathbf{E}(\mathbf{r},t).$$

The permittivity $\varepsilon(\mathbf{r},t)$ describes optical properties of the random medium.

Let the incident electromagnetic field be a plane monochromatic wave with wavelength λ and frequency ω. We neglect a permittivity variation during the time it takes the wave to propagate through the system, since this time is much shorter than that of a Brownian particle shift at a wavelength distance. In this case Eq. (2.4) can be presented in the integral form

$$\mathbf{E}(\mathbf{r},t)=\langle\mathbf{E}(\mathbf{r},t)\rangle+\int d\mathbf{r}_1\hat{T}(\mathbf{r}-\mathbf{r}_1)\frac{\delta\varepsilon(\mathbf{r}_1,t)}{4\pi}\mathbf{E}(\mathbf{r}_1,t), \quad (2.5)$$

where $\langle\mathbf{E}(\mathbf{r},t)\rangle$ is the mean field and $\hat{T}(\mathbf{r})$ is the electromagnetic field propagator taking the form within the far zone approximation

$$T_{\alpha\beta}(\mathbf{r})=\frac{k_0^2}{r}\left(\delta_{\alpha\beta}-\frac{r_\alpha r_\beta}{r^2}\right)e^{ikr}. \quad (2.6)$$

Here $k_0=\omega/c$, $k=\sqrt{\varepsilon}k_0$ is the wave number in the medium, and $\varepsilon=\langle\varepsilon(\mathbf{r},t)\rangle$ is the mean permittivity. Permittivity ε contains an imaginary part due to light losses during scattering. Therefore wave number k also contains an imaginary part determining the extinction length $l=(2Im\ k)^{-1}$.

Equation (2.5) is solved by iterations. Multiplying two such iterative solutions obtained for moments 0 and t, respectively, and averaging the product, we obtain the field correlation function within the ladder approximation

$$\begin{aligned}C_E(t)&=\langle\delta\mathbf{E}(\mathbf{r}_0,0)\,\delta\mathbf{E}^*(\mathbf{r}_0,t)\rangle\\&=\int d\mathbf{r}_1 d\mathbf{r}_1'\hat{T}(\mathbf{r}_0-\mathbf{r}_1)\hat{T}^*(\mathbf{r}_0-\mathbf{r}_1')G(\mathbf{r}_1-\mathbf{r}_1',t)\mathbf{E}(\mathbf{r}_1,0)\\&\quad\times\mathbf{E}^*(\mathbf{r}_1',t)+\sum_{n=1}^{\infty}\int d\mathbf{r}_{n+1}d\mathbf{r}_{n+1}'\hat{T}(\mathbf{r}_0-\mathbf{r}_{n+1})\\&\quad\times\hat{T}^*(\mathbf{r}_0-\mathbf{r}_{n+1}')G(\mathbf{r}_{n+1}-\mathbf{r}_{n+1}',t)\prod_{l=1}^{n}\int d\mathbf{r}_l d\mathbf{r}_l'\\&\quad\times\hat{T}(\mathbf{r}_{l+1}-\mathbf{r}_l)\hat{T}^*(\mathbf{r}_{l+1}'-\mathbf{r}_l')G(\mathbf{r}_l-\mathbf{r}_l',t)\mathbf{E}(\mathbf{r}_1,0)\\&\quad\times\mathbf{E}^*(\mathbf{r}_1',t),\end{aligned} \quad (2.7)$$

where

$$G(\mathbf{r}_l-\mathbf{r}_l',t)=\frac{1}{(4\pi)^2}\langle\delta\varepsilon(\mathbf{r}_l,0)\,\delta\varepsilon(\mathbf{r}_l',t)\rangle, \quad (2.8)$$

and $\delta\varepsilon(\mathbf{r}_l,0)$ and $\delta\varepsilon(\mathbf{r}_l',t)$ are permittivity fluctuations at moments 0 and t, respectively. We omit factors $\exp(\pm i\omega t)$ describing the temporal dependence of the incident monochromatic wave, since they cancel out when transiting to the intensity correlation function (2.1).

We define the Fourier transform of the permittivity correlation function $G(\mathbf{r},t)$ as follows:

$$G(\mathbf{r},t)=\int\frac{d^3\mathbf{q}}{(2\pi)^3}\tilde{G}(\mathbf{q},t)\exp(i\mathbf{q}\cdot\mathbf{r}). \quad (2.9)$$

Let the scattering system be an ensemble of particles undergoing the Brownian motion. In this case one can present

$$\tilde{G}(\mathbf{q},t)=G_0(\mathbf{q})\exp(-D_s q^2 t), \quad (2.10)$$

where $G_0(\mathbf{q})$ is the Fourier transform of the static correlation function, and D_s is the self-diffusion coefficient. We consider the dispersion of static correlations being negligible, i.e., $G_0(\mathbf{q})=G_0$. This is tantamount to assuming on a small size of the scatterers, $r_c\ll\lambda$, where r_c is either the permittivity correlation length or the scatterer size.

III. TEMPORAL CORRELATION FUNCTION FOR BACKSCATTERING. SCALAR FIELD

We assume the heterogeneous medium to occupy a half-space with boundary $z=0$, where z is a Cartesian coordinate, directed inward toward the medium. We consider backscattering at a small angle θ counted from the backward direction. To avoid considering a cyclic diagram contribution, in this section we assume angle θ to be outside the narrow coherent backscattering cone. Two of these conditions determine the angular interval $\lambda/l \ll \theta \ll 1$.

Let the radiation fall normally upon an illuminated area $A = W^2$ in the $z=0$ plane. The incident wave vector can be presented in Cartesian components as follows:

$$\mathbf{k}_i = \mathbf{k}_i' + i\mathbf{k}_i'', \quad \mathbf{k}_i' = (0,0,k), \quad \mathbf{k}_i'' = (0,0,\sigma/2), \tag{3.1}$$

where $\sigma = l^{-1}$ is the turbidity. The effective depth of radiation traveling inside the medium is of the order of l. One assumes a distance $|\mathbf{r}_0 - \mathbf{r}_1|$ to the observation point $\mathbf{r}_0$ to exceed significantly the linear dimensions of illuminated volume $V = Al$, i.e., $r_0 \gg W$ and l.

Propagating through a highly heterogeneous medium, light usually becomes depolarized due to the multiple scattering. Therefore one generally describes effects stemming from multiple scattering considering a scalar field instead of the real electromagnetic one. In this case the electric field in Eq. (2.7) is changed to a scalar, and the propagator $\hat{T}(\mathbf{r})$ transits to $T(\mathbf{r}) = r^{-1} k_0^2 \exp(ikr)$ with the Fourier transform

$$T(\mathbf{r}) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \exp(i\mathbf{p}\cdot\mathbf{r}) \widetilde{T}(\mathbf{p}), \quad \widetilde{T}(\mathbf{p}) = \frac{4\pi k_0^2}{p^2 - k^2}, \tag{3.2}$$

The pair of propagators containing the observation point takes the form

$$T(\mathbf{r}_0 - r_n) T^*(\mathbf{r}_0 - \mathbf{r}_n') \approx \frac{k_0^4}{r_0^2} \exp[-i\mathbf{k}_s \cdot (\mathbf{r}_n - \mathbf{r}_n')], \tag{3.3}$$

where $\mathbf{k}_s = k\mathbf{r}_0 / r_0$ is the wave vector of the scattered wave directed to the observation point. For backward scattering, one has

$$\mathbf{k}_s = \mathbf{k}_s' + i\mathbf{k}_s'', \quad \mathbf{k}_s' \approx (0,0,-k), \quad \mathbf{k}_s'' \approx (0,0,-\sigma/2). \tag{3.4}$$

The permittivity correlation function $G(\mathbf{r}_l - \mathbf{r}_l', t)$ of Eq. (2.8) does not vanish only for distances $|\mathbf{r}_l - \mathbf{r}_l'| \leqslant r_c$. Therefore, taking into account $r_c \ll l$ in exponentials containing initial or scattered wave vectors, one obtains

$$\exp(i\mathbf{k}_i \cdot \mathbf{r}_1 - i\mathbf{k}_i^* \cdot \mathbf{r}_1') \approx \exp[i\mathbf{k}_i' \cdot (\mathbf{r}_1 - \mathbf{r}_1{}') - \sigma z_1], \tag{3.5}$$

$$\exp(-i\mathbf{k}_s \cdot \mathbf{r}_n + i\mathbf{k}_s^* \cdot \mathbf{r}_n') \approx \exp[-i\mathbf{k}_s' \cdot (\mathbf{r}_n - \mathbf{r}_n') - \sigma z_n].$$

As is seen from these equations, the integrals with respect to $\mathbf{r}_1'$ and $\mathbf{r}_{n+1}'$ in Eq. (2.7) can be taken over infinite space due to the boundedness of the permittivity correlation function. The spatially restricted character of the system affects only the integrations with respect to $\mathbf{r}_1$ and $\mathbf{r}_{n+1}$. The integration with respect to $\mathbf{r}_1$ yields

$$\int_{z_1 \geqslant 0} d\mathbf{r}_1 \exp(i\mathbf{k}_i' \cdot \mathbf{r}_1 - \sigma z_1) G(\mathbf{r}_1 - \mathbf{r}_1', t) T(\mathbf{r}_2 - \mathbf{r}_1)$$
$$= \int\int \frac{d\mathbf{q}\, d\mathbf{q}_1}{(2\pi)^6} \widetilde{G}(\mathbf{q}_1, t) \widetilde{T}(\mathbf{p}_1 + \mathbf{q})$$
$$\times \exp[i\mathbf{q}_1 \cdot \mathbf{r}_1' + i(\mathbf{p}_1 + \mathbf{q}) \cdot \mathbf{r}_2] A_1,$$

where

$$A_1 = \int_{z_1 \geqslant 0} d\mathbf{r}_1 \exp[i\mathbf{r}_1 \cdot (\mathbf{k}_i' - \mathbf{q}_1 - \mathbf{p}_1 - \mathbf{q}) - \sigma z_1].$$

Using the momentum conservation law $\mathbf{k}_i' - \mathbf{p}_1 - \mathbf{q}_1 = 0$, we calculate

$$A_1 = \frac{(2\pi)^2 \delta_2(\mathbf{q}_\perp)}{\sigma + iq_z}, \tag{3.6}$$

where $\mathbf{q}_\perp$ is the transversal component of wave vector $\mathbf{q}$.

Similarly the integral over $\mathbf{r}_{n+1}$ yields

$$A_2 = \int_{z_{n+1} \geqslant 0} d\mathbf{r}_{n+1} \exp(i\mathbf{r}_{n+1} \cdot \mathbf{q} - \sigma z_{n+1}) = \frac{(2\pi)^2 \delta_2(\mathbf{q}_\perp)}{\sigma - iq_z}. \tag{3.7}$$

Multiplying A_1 and A_2, one of two delta functions $\delta_2(\mathbf{q}_\perp)$ is replaced by the illuminated area A, $\delta_2(\mathbf{q}_\perp) \to A/(2\pi)^2$. As a result we obtain

$$A_1 A_2 = \frac{(2\pi)^2 A \delta_2(\mathbf{q}_\perp)}{q_z^2 + \sigma^2}. \tag{3.8}$$

Performing Fourier transformation of functions $G(\mathbf{r},t)$ and $T(\mathbf{r})$, we present the field correlation function for backscattering $C_E^{(R)}(t)$ as follows:

$$C_E^{(R)}(t) = B \int_{-\infty}^{\infty} \frac{dq_z}{q_z^2 + \sigma^2} \Bigg\{ \exp[-D_s t(\mathbf{k}_i - \mathbf{k}_s)^2]$$
$$+ \sum_{n=1}^{\infty} \left(\frac{\sigma}{2\pi^2} \right)^n \int d\mathbf{p}_1 d\mathbf{p}_2 \ldots d\mathbf{p}_n$$
$$\times \exp\{-D_s t[(\mathbf{k}_i - \mathbf{p}_1)^2 + (\mathbf{p}_1 - \mathbf{p}_2)^2 + \cdots$$
$$+ (\mathbf{p}_{n-1} - \mathbf{p}_n)^2 + (\mathbf{p}_n - \mathbf{k}_s)^2]\}$$
$$\times \prod_{l \leqslant n} \frac{1}{[(\mathbf{p}_l + \mathbf{q})^2 - k^2](p_l^2 - k^{*2})} \Bigg\}, \tag{3.9}$$

where $B \sim AG_0$ is a multiplicative coefficient whose value is unessential for what follows.

Deriving Eq. (3.9) from the scalar analog of Eq. (2.7), we take spatial integrals over unrestricted space except for those ascribed either to the first or the latter scattering events. Such an approximation is known [3,19] to bring to a divergent result at $t=0$. Thus boundary conditions are to be taken into account properly [20]. The mirror image method is used for this purpose [16,19–21], which leads to a substitution

$$\frac{1}{q_z^2+\sigma^2} \to \frac{1}{\sigma^2} f(w), \tag{3.10}$$

where $w = q_z/\sigma$. A specific form of function $f(w)$ depends on the choice of a mirror image plane. Taking the latter as $z = z_b$, $z_b = -0.7104 l$, in correspondence with the classic solution of the Milne problem (see [20]), one obtains

$$f(w) = \frac{(1-w^2)[1-\cos(1.4208\,w)] + 2w[w+\sin(1.4208\,w)]}{(1+w^2)^2}. \tag{3.11}$$

Equation (3.9) describing the temporal correlation function as the series in scattering orders is valid for any value of t. We restrict ourselves to the small time limit, $t/\tau \ll 1$, where $\tau = (D_s k^2)^{-1}$ is the characteristic time it takes a Brownian particle to diffuse a wavelength distance. The $p_l = k$ neighborhood makes the main contribution to the threefold integrals with respect to $\mathbf{p}_l$ due to the pole of the integrand. Changing to a spherical coordinate frame

$$\int d\mathbf{p}_l = \int_0^\infty p_l^2 dp_l \int d\Omega_l ,$$

and expanding the integrand in partial fractions we obtain

$$\int_0^\infty \frac{p_l^2 dp_l}{[(\mathbf{p}_l+\mathbf{q})^2-k^2][p_l^2-k^{*2}]} \approx \frac{\pi}{2(\sigma+iq_z\cos\theta_l)}, \tag{3.12}$$

where θ_l is the angle between vector $\mathbf{p}_l$ and axis z. Calculating this integral we took into account that q_z is of order of σ and is hence significantly less than $|k|$.

Thus angular integrals over orientations of vectors $\mathbf{p}_l$ remain to be calculated in Eq. (3.9),

$$C_E^{(R)}(t) = B\int_{-\infty}^{\infty} \frac{1}{\sigma^2} f(w)\, dq_z \Bigg\{ \exp\left(-\frac{4t}{\tau}\right) + \sum_{n=1}^{\infty} \left(\frac{\sigma}{4\pi}\right)^n \times \prod_{l\leq n} \int \frac{d\Omega_l}{\sigma+iq_z\cos\theta_l} \exp\Bigg\{-2\frac{t}{\tau}\Bigg[(n+1) - \left(\frac{\mathbf{k}_i\cdot\mathbf{p}_1}{k^2} + \frac{\mathbf{p}_1\cdot\mathbf{p}_2}{k^2} + \cdots + \frac{\mathbf{p}_{n-1}\cdot\mathbf{p}_n}{k^2} + \frac{\mathbf{p}_n\cdot\mathbf{k}_s}{k^2}\right)\Bigg]\Bigg\}\Bigg\}. \tag{3.13}$$

Expanding exponentials into series in order of t/τ, one obtains

$$\prod_{l\leq n} \int \frac{d\Omega_l}{\sigma+iq_z\cos\theta_l} \exp\left[\frac{2t}{\tau}\left(\frac{\mathbf{k}_i\cdot\mathbf{p}_1}{k^2} + \frac{\mathbf{p}_1\cdot\mathbf{p}_2}{k^2} + \cdots + \frac{\mathbf{p}_{n-1}\cdot\mathbf{p}_n}{k^2} + \frac{\mathbf{p}_n\cdot\mathbf{k}_s}{k^2}\right)\right]$$
$$= \prod_{l\leq n} \int \frac{d\Omega_l}{\sigma+iq_z\cos\theta_l} \Bigg\{ 1 + \frac{2t}{k^2\tau}(n-1)\mathbf{p}_1\cdot\mathbf{p}_2 + \left(\frac{2t}{\tau}\right)^2 \times \frac{1}{2k^4}[2(\mathbf{k}_i\cdot\mathbf{p}_1)^2 - 2(\mathbf{k}_i\cdot\mathbf{p}_1)(\mathbf{k}_i\cdot\mathbf{p}_2) + (n-1)(\mathbf{p}_1\cdot\mathbf{p}_2)^2 + 2(n-2)(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{p}_2\cdot\mathbf{p}_3) + (n-2)(n-3) \times (\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{p}_3\cdot\mathbf{p}_4)] + \cdots \Bigg\}. \tag{3.14}$$

Deriving Eq. (3.14), the equality $\mathbf{k}_i = -\mathbf{k}_s$, valid for the backward direction, as well as a symmetry of the integrand with respect to the $\mathbf{p}_l$ permutation, were used.

Now these angular integrals in Eq. (3.14) are calculated explicitly,

$$\int \frac{d\Omega_1}{\sigma+iq_z\cos\theta_1} = \frac{4\pi}{\sigma} p_0 ,$$

$$\frac{1}{k^2}\int \frac{d\Omega_1(\mathbf{k}_i\cdot\mathbf{p}_1)}{\sigma+iq_z\cos\theta_1} = -\frac{4\pi i}{\sigma} w p_1 ,$$

$$\frac{1}{k^4}\int \frac{d\Omega_1(\mathbf{k}_i\cdot\mathbf{p}_1)^2}{\sigma+iq_z\cos\theta_1} = \frac{4\pi}{\sigma} p_1 ,$$

$$\frac{1}{k^2}\int \frac{d\Omega_1 d\Omega_2(\mathbf{p}_1\cdot\mathbf{p}_2)}{(\sigma+iq_z\cos\theta_1)(\sigma+iq_z\cos\theta_2)} = -\left(\frac{4\pi}{\sigma}\right)^2 w^2 p_1^2 ,$$

$$\frac{1}{k^4}\int \frac{d\Omega_1 d\Omega_2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{(\sigma+iq_z\cos\theta_1)(\sigma+iq_z\cos\theta_2)} = \left(\frac{4\pi}{\sigma}\right)^2 \left[\frac{1}{2}p_0^2 - p_0 p_1 + \frac{3}{2}p_1^2\right],$$

$$\frac{1}{k^4}\int \frac{d\Omega_1 d\Omega_2 d\Omega_3(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{p}_2\cdot\mathbf{p}_3)}{(\sigma+iq_z\cos\theta_1)(\sigma+iq_z\cos\theta_2)(\sigma+iq_z\cos\theta_3)} = -\left(\frac{4\pi}{\sigma}\right)^3 w^2 p_1^3 , \tag{3.15}$$

where auxiliary functions are introduced:

$$p_0 = p_0(w) = \frac{1}{w}\arctan w,$$

$$p_1 = p_1(w) = \frac{1}{w^2}(1 - p_0). \tag{3.16}$$

Substituting Eqs. (3.14) and (3.15) into Eq. (3.13) we come to the expression

$$C_E^{(R)}(t) = \frac{B}{\sigma}\int_{-\infty}^{\infty} dw f(w)\Bigg(\exp\left(-\frac{4t}{\tau}\right)(1+p_0) + \sum_{n=2}^{\infty}\exp\left[-\frac{2t}{\tau}(n+1)\right] \times\Bigg\{p_0^n - \frac{2t}{\tau}w^2(n-1)p_1^2p_0^{n-2} + \frac{1}{2}\left(\frac{2t}{\tau}\right)^2[2p_1p_0^{n-1} + 2w^2p_1^2p_0^{n-2} + (n-1)p_0^{n-2} \times(\tfrac{3}{2}p_1^2 - p_0p_1 + \tfrac{1}{2}p_0^2) - 2(n-2)w^2p_1^3p_0^{n-3} + (n-2)(n-3)w^4p_1^4p_0^{n-4}] + \cdots\Bigg\}\Bigg). \tag{3.17}$$

Series of separate terms within inner curly brackets are easily summed as geometric progressions and their derivatives with respect to parameter p_0 to yield the correlation function as follows:

$$C_E^{(R)}(t) = B\frac{\exp\left(-\frac{4t}{\tau}\right)}{\sigma}\int_{-\infty}^{\infty} f(w)dw\Bigg\{1 + p_0\phi(t) - \frac{4t}{\tau}p_1^2w^2 \times\exp\left(-\frac{2t}{\tau}\right)\phi^2(t) + \left(\frac{2t}{\tau}\right)^2 \times\phi(t)\Bigg[p_1\left(1 + w^2\exp\left(-\frac{2t}{\tau}\right)\right) + \tfrac{1}{2}(\tfrac{3}{2}p_1^2 - p_0p_1 + \tfrac{1}{2}p_0^2)\exp\left(-\frac{2t}{\tau}\right)\phi(t) + w^2p_1^3 \times\exp\left(-\frac{4t}{\tau}\right)\phi(t) \times\left(-1 + w^2p_1\exp\left(-\frac{2t}{\tau}\right)\phi(t)\right)\Bigg] + \cdots\Bigg\}, \tag{3.18}$$

where function $\phi(t) = [1 - p_0\exp(-2t/\tau)]^{-1}$ stems from the infinite number of terms $p_0^n\exp(-2tn/\tau)$ in series (3.17).

We calculate the temporal field correlation function $C^{(R)}(t)$ for backscattered radiation from Eq. (3.18) using no adjustable parameters. The results are shown in Fig. 1. The temporal correlation function is known [1–3] to depend linearily on $\sqrt{t/\tau}$ in the small time range. For this reason we plot the correlation function against this temporal variable. From Fig. 3 of Ref. [5] we show measurement data [2] for the temporal correlation function of light scattered from a polysterene latex suspension, and a theoretical interpolation plot proposed there. As is seen from the plots, our results agree rather well with the observed behavior. For larger values of t, $\sqrt{t/\tau} \geqslant 0.5$, the curve resulting from Eq. (3.18) appears to be closer to experimental data than the interpolation of Ref. [5]. Describing an initial decay rate of the temporal correlation function at $t \ll \tau$, one defines a slope γ as a coefficient in the relationship

$$C_E^{(R)}(t) \approx [1 + \gamma(6t/\tau)^{1/2}]^{-1}. \tag{3.19}$$

The calculation yields $\gamma = 1.9$ as compared with the value $\gamma \approx 2$ obtained from experiment [2]. For comparison we also show curve (3.19) with $\gamma = 2$ used as a fit in Ref. [2].

Changing $\phi(t)$ to

$$\phi_n(t) = \frac{1 - [p_0\exp(-2t/\tau)]^{n+1}}{1 - p_0\exp(-2t/\tau)} \tag{3.20}$$

in Eq. (3.18), one obtains an expression describing a contribution of n scattering orders to the correlation function in the small time limit $t \ll \tau$. We calculate the temporal correlation function $C_n^{(R)}(t)$ arising from the n scattering order contribution and plot it in Fig. 2. Calculated results are compared with experimental data [2] for backscattering from slabs with different thicknesses $L = 0.6$, 1, and 2 mm. The main contribution is assumed to be given into the correlation function by the terms of series (3.17) up to $n = 2L/l^*$, where l^* is the transport mean free path. Taking $l^* = 144\ \mu$m [2], we choose $n = 8$, 14, and 28. The calculated plot is seen to roll over at short times quite similar to the experimental one. The magnitude of deviation from a straight line also increases with L correspondingly to measurements. A similar accumulation of scattering order inputs into backscattering enhancement was analyzed in Ref. [20].

IV. POLARIZATION EFFECTS IN BACKSCATTERING

In this section we take into account the vector nature of an electromagnetic field, and calculate the intensity correlation functions for polarized,

$$C_I^{VV}(t) = \langle \delta I_V^V(0)\,\delta I_V^V(t)\rangle - \langle \delta I_V^V\rangle^2, \tag{4.1}$$

and depolarized,

$$C_I^{VH}(t) = \langle \delta I_H^V(0)\,\delta I_H^V(t)\rangle - \langle \delta I_H^V\rangle^2, \tag{4.2}$$

components of scattered light. Within the factorization approximation they are presented as products of the field correlation functions,

$$C_I^{VV}(t) = |C_E^{VV}(t)|^2, \quad C_I^{VH}(t) = |C_E^{VH}(t)|^2. \tag{4.3}$$

Since the vector nature of the field brings about different angular dependences of polarized and depolarized components, we consider a contribution of cyclic diagrams along with that of ladder ones. This permits a simultaneous description of temporal and angular behavior of the correlation function for backscattered light. To avoid cumbersome derivation, we restrict ourselves to the small time limit $t \ll \tau$.

Let the light wave fall normally upon the $z = 0$ boundary and be polarized along axis y, $\mathbf{E}_i = (0, E, 0)$. The scattered light is observed in the (x,z) plane, with the wave vector defined as $\mathbf{k}_s=(k_0\theta,0,-k)$. Ignoring a projection δE_z of scattered field on the z axis at small angle θ, we define the field correlation functions for polarized and depolarized components as follows:

$$C_E^{VV}(t)=\langle \delta E_y(0)\,\delta E_y(t)\rangle,\qquad C_E^{VH}(t)=\langle \delta E_x(0)\,\delta E_x(t)\rangle. \tag{4.4}$$

Since $r_c \ll \lambda$ within the pointlike scatterer approximation, we set coordinates of complex-conjugated propagators to be equal pairwise, and introduce, for convenience, the fourth-rank tensor

$$T_{\alpha\beta}(\mathbf{r})T^*_{\gamma\delta}(\mathbf{r})=k_0^4\Lambda_{\alpha\gamma,\beta\delta}(\mathbf{r}). \tag{4.5}$$

At small times the value of the wave-vector transfer $\mathbf{q}$ entering the fluctuation correlator $G(\mathbf{q})$ is known to be changed to its mean value $q^2\to\langle q^2\rangle=2k^2$ [1]. Indeed, every function $\widetilde{G}(\mathbf{q},t)$ in Eq. (3.16) brings a factor $\exp(-2t/\tau)\sim 1-2D_sk^2t$ at $t\ll\tau$.

Thus contributions of ladder and cyclic diagrams to the field correlation function can be presented as follows:

$$\langle \delta E_\alpha(\mathbf{r}_0,0)\,\delta E^*_\alpha(\mathbf{r}_0,t)\rangle\sim\exp(-4t/\tau)\times[G^{(L)}_{\alpha\alpha,yy}(t)+G^{(C)}_{\alpha\alpha,yy}(t)], \tag{4.6}$$

where

$$G^{(L)}_{\alpha\alpha,yy}(t)=k_0^4\int d\mathbf{r}_1 d\mathbf{r}_2\exp[-\sigma(z_1+z_2)]\times\Big[\hat\Lambda(\mathbf{r}_1-\mathbf{r}_2)+k_0^4G_0\exp(-2t/\tau)\times\int d\mathbf{r}_3\hat\Lambda(\mathbf{r}_1-\mathbf{r}_3)\hat\Lambda(\mathbf{r}_3-\mathbf{r}_2)+\cdots\Big]_{\alpha\alpha,yy} \tag{4.7}$$

and

$$G^{(C)}_{\alpha\alpha,yy}(t)=k_0^4\int d\mathbf{r}_1 d\mathbf{r}_2\exp[-\sigma(z_1+z_2)+i(\mathbf{r}_2-\mathbf{r}_1)(\mathbf{k}_i+\mathbf{k}_s)]\Big[\hat\Lambda(\mathbf{r}_1-\mathbf{r}_2)+k_0^4G_0\times\exp(-2t/\tau)\int d\mathbf{r}_3\hat\Lambda(\mathbf{r}_1-\mathbf{r}_3)\times\hat\Lambda(\mathbf{r}_3-\mathbf{r}_2)+\cdots\Big]_{\alpha\alpha,yy}. \tag{4.8}$$

Summing over index α is not assumed here. Spatial integrals are to be taken over the volume of the scattering system. As is seen one has to sum the same series in Eqs. (4.7) and (4.8). Denoting the sought sum of series as $\hat S(\mathbf{r}_1,\mathbf{r}_2,t)$ and using a standard procedure of summation, we obtain the well-known Dyson-like equation

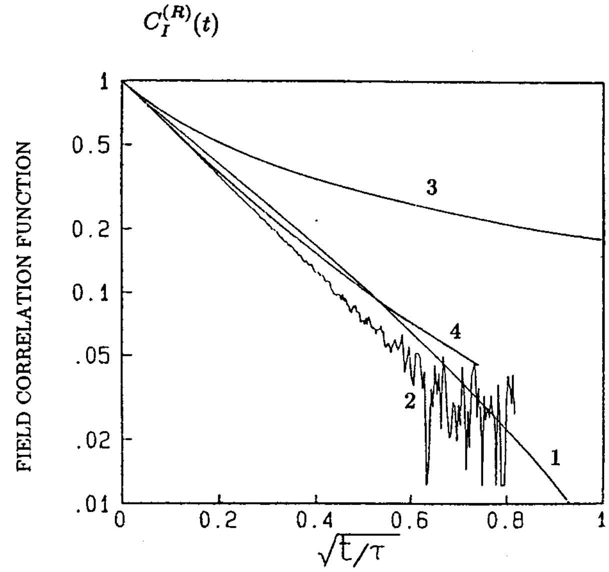

FIG. 1. Field correlation functions vs square root of time: curve 1 results from Eq. (3.18), curve 2 represents measurement data [2,5], curve 3 is a fit (3.19) with $\gamma=2$, and curve 4 results from a theoretical interpolation of Ref. [5].

$$\hat S(\mathbf{r}_1,\mathbf{r}_2,t)=\hat\Lambda(\mathbf{r}_1-\mathbf{r}_2)+\xi\int d\mathbf{r}_3\hat\Lambda(\mathbf{r}_1-\mathbf{r}_3)\hat S(\mathbf{r}_3,\mathbf{r}_2,t), \tag{4.9}$$

where $\xi=k_0^4G_0\exp(-2t/\tau)$. Function $\hat S(\mathbf{r}_1,\mathbf{r}_2,t)$ is known to be an average of the product of two Green functions of wave equation (2.4) less the single-scattering contribution, and can be termed as the radiative transfer propagator. In the case of a restricted system $\hat S(\mathbf{r}_1,\mathbf{r}_2,t)$ becomes translationally non-

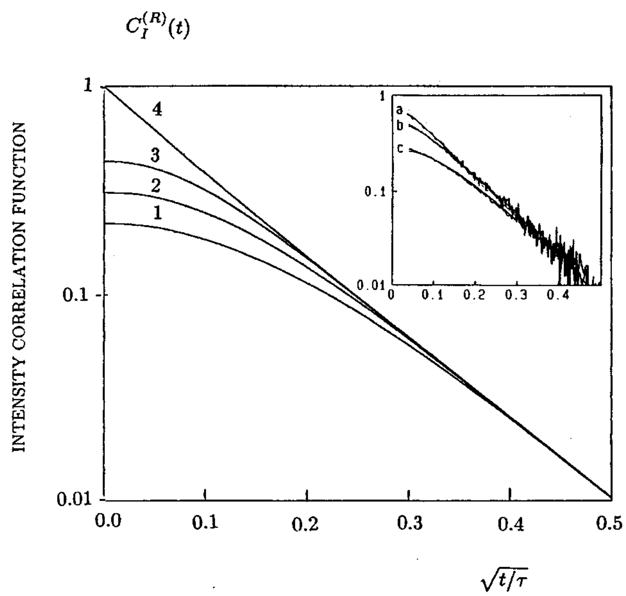

FIG. 2. Contribution of n scattering orders to intensity correlation function $C_I^{(R)}(t)$ vs square root of time: curve 1, $n=8$; curve 2, $n=14$; curve 3, $n=28$; curve 4, total contribution. The inset represents experimental plots [2] for backscattering from slabs with different thicknesses: (a) $L=2$ mm. (b) $L=1$ mm. (c) $L=0.6$ mm.

invariant, as distinct from the priming propagator $\hat{\Lambda}(\mathbf{r}_1-\mathbf{r}_2)$. Using the mirror image method for light scattering from the $z>0$ half-space, one presents the propagator $\hat{S}(\mathbf{r}_1,\mathbf{r}_2,t)$ in the form

$$\hat{S}(\mathbf{r}_1,\mathbf{r}_2,t)=\hat{S}^{(0)}(\mathbf{r}_1-\mathbf{r}_2,t)-\hat{S}^{(0)}(x_1-x_2,y_1-y_2,z_1+z_2-2z_b,t), \tag{4.10}$$

where $\hat{S}^{(0)}(\mathbf{r},t)=\hat{S}^{(0)}(x,y,z,t)$ is the radiative transfer propagator for an infinite medium, and point $\mathbf{r}_b=(x,y,-z+2z_b)$ is the mirror image of $\mathbf{r}=(x,y,z)$ with respect to the $z=z_b$ plane. Equation (4.10) guarantees that propagator $\hat{S}(\mathbf{r}_1,\mathbf{r}_2,t)$ is zero if at least one of two points $\mathbf{r}_1,\mathbf{r}_2$ is placed in the $z=z_b$ plane.

Closing Eqs. (4.7) and (4.8) by means of function $\hat{S}(\mathbf{r}_1,\mathbf{r}_2,t)$, and using approximation (4.10) we present the field correlation function as

$$\langle \delta E_\alpha(\mathbf{r}_0,0)\,\delta E^*_\alpha(\mathbf{r}_0,t)\rangle \sim \exp(-4t/\tau)\times\int_{-\infty}^{\infty}\frac{dq_z}{2\pi}f(q_z)[\tilde{S}^{(0)}_{\alpha\alpha,yy}(q_z,t)+\tilde{S}^{(0)}_{\alpha y,y\alpha}(q,t)], \tag{4.11}$$

where $\alpha=x$ or y, $\hat{\tilde{S}}^{(0)}(\mathbf{q},t)=\int \hat{S}^{(0)}(\mathbf{r},t)e^{-i\mathbf{q}\cdot\mathbf{r}}d\mathbf{r}$ is the Fourier transform of the radiative transfer propagator, and $\mathbf{q}=(k_0\theta,0,q_z)$.

Accounting for the single-scattering contribution, the intensity correlation functions of polarized and depolarized scattered fields can be presented as follows:

$$C_E^{VV}(t)\sim\exp(-4t/\tau)\left\{1+3\int_{-\infty}^{\infty}\frac{dq_z}{2\pi}f(q_z)[\tilde{S}^{(0)}_{yy,yy}(q_z,t)+\tilde{S}^{(0)}_{yy,yy}(q,t)]\right\},$$

$$C_E^{VH}(t)\sim 3\exp(-4t/\tau)\int_{-\infty}^{\infty}\frac{dq_z}{2\pi}f(q_z)\times[\tilde{S}^{(0)}_{xx,yy}(q_z,t)+\tilde{S}^{(0)}_{xy,yx}(q,t)]. \tag{4.12}$$

Thus to solve the problem one has to find components of tensor $\hat{\tilde{S}}^{(0)}(\mathbf{q},t)$. Performing the Fourier transformation of Eq. (4.9) for an infinite homogeneous medium, we obtain

$$\tilde{S}^{(0)}_{\alpha\beta,\phi\psi}(q,t)=\tilde{\Lambda}_{\alpha\beta,\phi\psi}(q)+\xi\tilde{\Lambda}_{\alpha\beta,\gamma\nu}(q)\tilde{S}^{(0)}_{\gamma\nu,\phi\psi}(q,t), \tag{4.13}$$

where $\tilde{\Lambda}(\mathbf{q})$ is the Fourier transform of tensor $\hat{\Lambda}(\mathbf{r})$

$$\tilde{\Lambda}_{\alpha\beta,\phi\psi}(q)=\int\frac{d\mathbf{r}}{r^2}\left(\delta_{\alpha\phi}-\frac{r_\alpha r_\phi}{r^2}\right)\left(\delta_{\beta\psi}-\frac{r_\beta r_\psi}{r^2}\right)\times\exp(-i\mathbf{q}\cdot\mathbf{r}-\sigma r). \tag{4.14}$$

Using the axial symmetry of this expression with respect to vector $\mathbf{q}$, the components of tensor (4.14) are easily calculated in the coordinate frame with the z axis directed along vector $\mathbf{q}$. The number of tensor indices x or y is to be even in this frame due to the indicated symmetry. Since the total number of indices is four, the number of index z is even also. Thus any nonzero component $\tilde{\Lambda}_{\alpha\beta,\gamma\delta}(\mathbf{q})$ can contain only two pairs of different indices. The definition (4.14) immediately gives the symmetry properties

$$\tilde{\Lambda}_{\alpha\beta,\gamma\delta}(q)=\tilde{\Lambda}_{\gamma\delta,\alpha\beta}(q)=\tilde{\Lambda}_{\beta\alpha,\delta\gamma}(q)=\tilde{\Lambda}_{\gamma\beta,\alpha\delta}(q). \tag{4.15}$$

Using auxiliary functions (3.16) and defining supplementarily $p_2=w^{-2}(\frac{1}{3}-p_1)$, we find components of the priming tensor (4.14) as follows:

$$\begin{aligned}
&\tilde{\Lambda}_{11,11}=\tilde{\Lambda}_{22,22}=\frac{\pi}{2\sigma}(3p_0+2p_1+3p_2),\\
&\tilde{\Lambda}_{11,22}=\tfrac{1}{8}\tilde{\Lambda}_{33,33}=\frac{\pi}{2\sigma}(p_0-2p_1+p_2),\\
&\tilde{\Lambda}_{12,12}=\frac{\pi}{2\sigma}(p_0+6p_1+p_2),\quad \tilde{\Lambda}_{jj,33}=\frac{2\pi}{\sigma}(p_1-p_2),\\
&\tilde{\Lambda}_{j3,j3}=\frac{2\pi}{\sigma}(p_0-p_2),\quad j=1\ \text{and}\ 2.
\end{aligned} \tag{4.16}$$

Here indices 1, 2, and 3 denote components in the Cartesian frame with component 3 directed along vector $\mathbf{q}$. Solving the equation set (4.13) with respect to the radiative transfer propagator, within this coordinate frame we obtain

$$\tilde{S}^{(0)}_{11,11}=\tilde{S}^{(0)}_{22,22}=\frac{1}{2}\left[\frac{\tilde{\Lambda}_{11,11}-\tilde{\Lambda}_{11,22}}{1-\xi\tilde{\Lambda}_{11,11}+\xi\tilde{\Lambda}_{11,22}}+\frac{(1-\xi\tilde{\Lambda}_{33,33})(\tilde{\Lambda}_{11,11}+\tilde{\Lambda}_{11,22})+2\xi\tilde{\Lambda}^2_{11,33}}{(1-\xi\tilde{\Lambda}_{33,33})(1-\xi\tilde{\Lambda}_{11,11}-\xi\tilde{\Lambda}_{11,22})-2\xi^2\tilde{\Lambda}^2_{11,33}}\right],$$

$$\tilde{S}^{(0)}_{11,22}=\frac{\tilde{\Lambda}_{11,22}}{(1-\xi\tilde{\Lambda}_{11,11}-\xi\tilde{\Lambda}_{11,22})(1-\xi\tilde{\Lambda}_{11,11}+\xi\tilde{\Lambda}_{11,22})}+\frac{\xi\tilde{\Lambda}^2_{11,33}}{(1-\xi\tilde{\Lambda}_{11,11}-\xi\tilde{\Lambda}_{11,22})\{[1-\xi\tilde{\Lambda}_{11,11}-\xi\tilde{\Lambda}_{11,22}](1-\xi\tilde{\Lambda}_{33,33})-2\xi^2\tilde{\Lambda}^2_{11,33}\}}, \tag{4.17}$$

$$\tilde{S}^{(0)}_{1j,j1}=\frac{\tilde{\Lambda}_{11,jj}}{(1-\xi\tilde{\Lambda}_{1j,1j})^2-\xi^2\tilde{\Lambda}^2_{11,jj}}, \quad j=2 \text{ and } 3.$$

Obtaining the polarized and depolarized light correlation functions, we calculate the following components: $\tilde{S}^{(0)}_{yy,yy}\times(\mathbf{q},t)$, $\tilde{S}^{(0)}_{xx,yy}(\mathbf{q},t)$, and $\tilde{S}^{(0)}_{xy,yx}(\mathbf{q},t)$, due to Eq. (4.11). These laboratory frame components of tensor $\tilde{S}^{(0)}(\mathbf{q},t)$ are related to the components found in the coordinate frame fixed by vector $\mathbf{q}$ as follows:

$$\tilde{S}^{(0)}_{yy,yy}(q,t)=\tilde{S}^{(0)}_{11,11}(q,t),$$

$$\tilde{S}^{(0)}_{xx,yy}(q,t)=\tilde{S}^{(0)}_{11,22}(q,t) \quad \text{at} \quad \theta=0, \tag{4.18}$$

$$\tilde{S}^{(0)}_{xy,yx}(q,t)=\tilde{S}^{(0)}_{12,21}(q,t)\cos^2\phi+\tilde{S}^{(0)}_{13,31}(q,t)\sin^2\phi,$$

where $\phi=\arctan(k_0\theta/q_z)$. Component $\tilde{S}^{(0)}_{xx,yy}(q)$ is given for $\theta=0$, since it describes the ladder diagram contribution depending solely on q_z due to Eq. (4.11).

We analyze first an asymptotics of propagator $\hat{\tilde{S}}(\mathbf{q},t)$ at small $q\ll\sigma$, since it defines essentially an initial decrease of correlation function with time and scattering angle. At $w=q/\sigma\ll 1$ functions p_n can be presented as

$$p_n=\frac{1}{2n+1}-\frac{w^2}{2n+3}+\frac{w^4}{2n+5}-\cdots \quad \text{for} \quad n=0,1,2 \ldots . \tag{4.19}$$

Substituting Eq. (4.19) into Eq. (4.16), we obtain asymptotics of the $\tilde{\Lambda}$ tensor components at small w

$$\tilde{\Lambda}_{11,11}=8\tilde{\Lambda}_{jj,33}=8\tilde{\Lambda}_{13,31}=\frac{4\pi}{\sigma}\left(\frac{8}{15}-\frac{8}{35}w^2\right),$$

$$\tilde{\Lambda}_{33,33}=8\tilde{\Lambda}_{11,22}=8\tilde{\Lambda}_{12,21}=\frac{4\pi}{\sigma}\left(\frac{8}{15}-\frac{8}{105}w^2\right), \tag{4.20}$$

$$\tilde{\Lambda}_{13,13}=\frac{4\pi}{\sigma}\left(\frac{2}{5}-\frac{2}{21}w^2\right),$$

$$\tilde{\Lambda}_{12,12}=\frac{4\pi}{\sigma}\left(\frac{2}{5}-\frac{22}{105}w^2\right), \quad j=1 \text{ and } 2.$$

The optical theorem permits us to relate the expansion parameter ξ to the turbidity σ,

$$\xi=\frac{3\sigma}{8\pi}\exp\left(-2\frac{t}{\tau}\right). \tag{4.21}$$

Substituting Eq. (4.20) into Eq. (4.17) and accounting for Eq. (4.21), we obtain

$$\tilde{S}^{(0)}_{yy,yy}(q,t)=\tilde{S}^{(0)}_{xx,yy}(q,t)\approx\frac{8\pi}{3\sigma}\frac{1}{(q/\sigma)^2+6t/\tau},$$
$$\tilde{S}^{(0)}_{xy,yx}(q,t)\approx\frac{16\pi}{9\sigma}. \tag{4.22}$$

The components $\tilde{S}^{(0)}_{yy,yy}(q,t)$ and $\tilde{S}^{(0)}_{xx,yy}(q,t)$ are seen to be singular at $\{q,t\}\to 0$, and to coincide with corresponding expressions for the radiative transfer propagator obtained earlier for scalar field [3], and $\tilde{S}^{(0)}_{xy,yx}(q,t)$ is finite.

Substituting Eq. (4.22) into integrals (4.11) and calculating them by the residue theorem, we find

$$C_E^{VV}(t)\sim\frac{1}{(1+\sqrt{6t/\tau})^2}\left\{1+\frac{1}{\sqrt{6t/\tau}}\times\left[1-\exp\left(-\frac{2|z_b|}{l}\sqrt{6t/\tau}\right)\right]\right\}+\frac{1}{(1+\sqrt{6t/\tau+(k_0\theta/\sigma)^2})^2}\left\{1+\frac{1}{\sqrt{6t/\tau+(k_0\theta/\sigma)^2}}\times\left[1-\exp\left(-\frac{2|z_b|}{l}\sqrt{6t/\tau+(k_0\theta/\sigma)^2}\right)\right]\right\}+B_1,$$

$$C_E^{VH}(t)\sim\frac{1}{(1+\sqrt{6t/\tau})^2}\times\left\{1+\frac{1}{\sqrt{6t/\tau}}\left[1-\exp\left(-\frac{2|z_b|}{l}\sqrt{6t/\tau}\right)\right]\right\}+B_2, \tag{4.23}$$

where B_1 and B_2 are terms, contributed from nonasymptotic parts of propagator, dropped in Eq. (4.22). These terms are known to have an analytic dependence on t and θ^2. Equations (4.23) predict the slope γ for the polarized component to depend on the scattering angle decreasing with increasing θ. At $\theta=0$ the slopes of polarized and depolarized components take the same value, $\gamma=2.4$, assuming that terms B_1 and B_2 are dropped.

In Fig. 3 the field correlation functions of polarized and depolarized scattered light components calculated from Eqs. (4.17) and (4.18) are plotted against $\sqrt{t/\tau}$ for different angles. For convenience all the quantities plotted were normalized to the polarized light intensity $\langle\delta I_V^V\rangle=C_E^{VV}(0)$ at $\theta=0$. We choose the dimensionless angular variable $Y=k_0 l\theta$ to be $Y=0$, 0.2, 1, and 2. For $\lambda=0.515$ μm and $l^*=19$ μm taken from Ref. [21], these Y values correspond to scattering angles $\theta=0$, 0.05, 0.25, and 0.5, in degrees, describing, in correspondence with measurement data

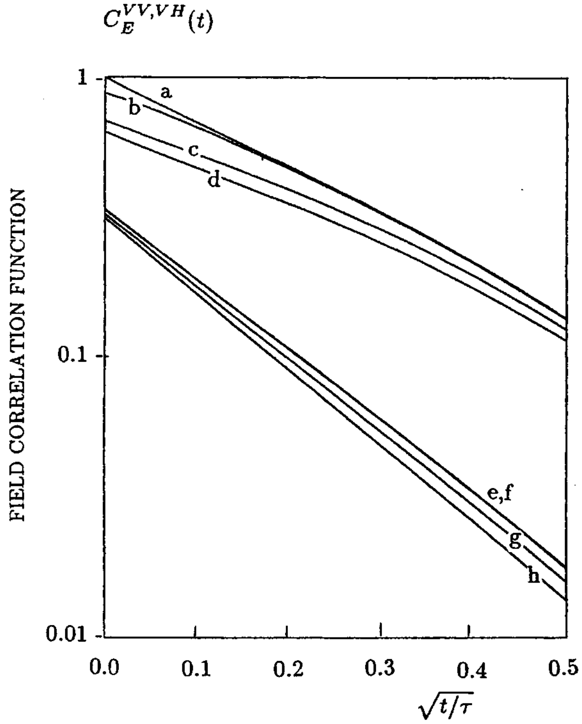

FIG. 3. Temporal dependence of field correlation function for polarized (curves a, b, c, and d) and depolarized (curves e, f, g, and h) components of scattered light at angular variable Y: curves a and e: $Y=0$; curves b and f: $Y=0.2$; curves c and g: $Y=1$; and curves d and h: $Y=2$.

[22], the temporal correlation function at the scattering peak, $\theta=0$ and 0.05, at half-width, $\theta=0.25$, and at the foot of the peak, $\theta=0.5$.

In Fig. 4 the polarized components with and without a single-scattering contribution (curves 1 and 2, respectively), and a depolarized one (curve 3), are shown for $Y\gg 1$, i.e., $\theta\gg\lambda/l$. To obtain results in this angular range, one is to account for the ladder diagram contribution only.

The polarized and depolarized components exhibit rather different temporal behaviors of the correlation function, the polarized one turning out to be more sensitive to the angle of scattering. The calculated slope $\gamma_{\rm pol}$ for the polarized component takes the values $\gamma_{\rm pol}$ = 1.5, 0.9, 1.1, and 1.2 for chosen angles, respectively, whereas the corresponding slope for the depolarized component varies with angle noticeably slighter, $\gamma_{\rm dep}$ = 2.2, 2.2, 2.3, and 2.4, respectively. At $Y\gg 1$ we get $\gamma_{\rm pol}=1.3$ and $\gamma_{\rm dep}=2.5$. Omitting the single-scattering contribution, we calculate $\gamma'_{\rm pol}=1.5$. These values are to be compared with $\gamma_{\rm pol}=1.6$ and $\gamma_{\rm dep}=2.6$ obtained in Ref. [5]. The slope of the polarized component exhibits a nonmonotonic angular behavior because the cyclic diagram contribution becomes regular with respect to t/τ at sufficiently small values of scattering angle involving a slope decrease, and then vanishes at large Y, increasing the slope once more. The results obtained permit a direct experimental verification.

V. TRANSMISSION CORRELATION FUNCTION

We consider temporal correlation function for radiation transmitted through a slab with thickness $L\gg l$, restricting

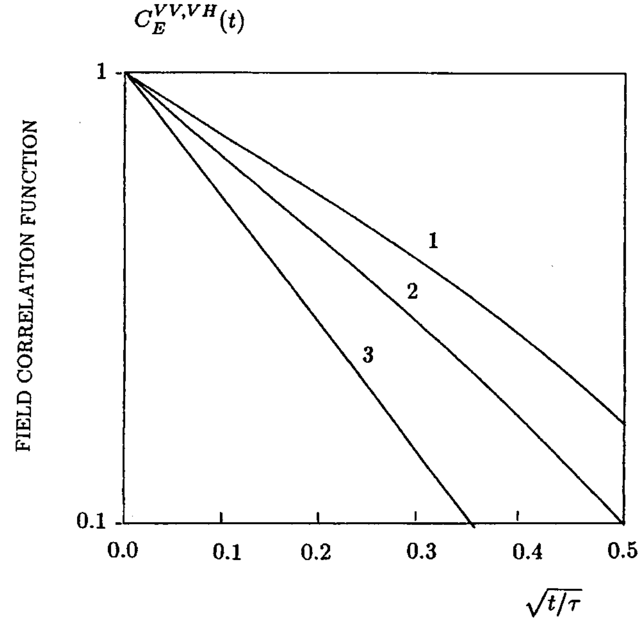

FIG. 4. Temporal dependence of the field correlation function outside the backscattering cone, $1\gg\theta\gg\lambda/l$: curve 1, polarized component; curve 2, polarized component less single-scattering contribution; curve 3, depolarized component.

ourselves to the scalar field case. Let a point, wherein the scattered radiation outgoing from the $z=L$ plane is observed, be written as $\mathbf{r}_0=(x_0,y_0,L+z_L)$ where z_L is a distance from the $z=L$ plane to the observation point along the normal. Defining vector $\mathbf{r}_L=(x_0,y_0,z_L)$, we obtain

$$|\mathbf{r}_0-\mathbf{r}_{n+1}|=\sqrt{(x_0-x_{n+1})^2+(y_0-y_{n+1})^2+(z_L+L-z_{n+1})^2}$$
$$\approx r_L-\frac{\mathbf{r}_L\cdot\mathbf{r}_{n+1}^{(L)}}{r_L}, \qquad (5.1)$$

where $\mathbf{r}_{n+1}^{(L)}=(x_{n+1},y_{n+1},z_{n+1}-L)$ is a medium point measured in a Cartesian frame with origin $(0,0,L)$. Then the pair of complex conjugated propagators containing the observation point can be written as

$$T(\mathbf{r}_0-\mathbf{r}_{n+1})T^*(\mathbf{r}_0-\mathbf{r}'_{n+1})\approx\left(\frac{k_0^2}{r_L}\right)^2\exp(-\mathbf{k}_s\cdot\mathbf{r}_{n+1}^{(L)}$$
$$+i\mathbf{k}_s^*\cdot\mathbf{r}'^{(L)}_{n+1}). \qquad (5.2)$$

One has $\mathbf{k}_s=\mathbf{k}_i$ for forward scattering. Taking into account relationship $|\mathbf{r}_l-\mathbf{r}'_l|\ll l$ and extracting imaginary parts of wave vectors, we get

$$\exp(-\mathbf{k}_s\cdot\mathbf{r}_{n+1}^{(L)}+i\mathbf{k}_s^*\cdot\mathbf{r}'^{(L)}_{n+1})$$
$$\approx\exp[-\mathbf{k}'_s\cdot(\mathbf{r}_{n+1}-\mathbf{r}'_{n+1})-\sigma(L-z_{n+1})]. \qquad (5.3)$$

Exponentials containing incident waves can be rearranged as

$$\exp(\mathbf{k}_i\cdot\mathbf{r}_1-i\mathbf{k}_i^*\cdot\mathbf{r}'_1)\approx\exp[i\mathbf{k}'_i\cdot(\mathbf{r}_1-\mathbf{r}'_1)-\sigma z]. \qquad (5.4)$$

Repeating arguments similar to those used when deriving Eq. (3.8), we obtain

$$A_1A_2=\frac{(2\pi)^2A\delta_2(\mathbf{q}_\perp)\exp(iq_zL)}{-(q_z-i\sigma)^2}\{1-\exp[-(\sigma+iq_z)L]\}^2. \tag{5.5}$$

Thus the transmission field correlation function can be written at $\sigma L \gg 1$ as a series in scattering orders

$$C_E^{(T)}(t)=-B\int_{-\infty}^{\infty}\frac{dq_z}{2\pi}\,\frac{\exp(iq_zL)}{(q_z-i\sigma)^2}\sum_{n=0}^{\infty}\left(\frac{\sigma}{2\pi^2}\right)^n \times\int d\mathbf{q}_1\cdots d\mathbf{q}_n\exp[-D_st(q_1^2+\cdots+q_{n+1}^2)] \times\prod_{l\leq n}\frac{1}{(p_l^2-k^{*2})[(\mathbf{p}_1+\mathbf{q})^2-k^2]}. \tag{5.6}$$

Equation (5.6) contains three length parameters: the scatterer diffusion path $l_t=(D_st)^{1/2}$, wavelength λ, and extinction length l. Accounting for $\lambda \ll l$ one separates three asymptotic regions $l_t \ll \lambda$, $\lambda \ll l_t \ll l$, and $l \ll l_t$ determining three temporal intervals wherein the analysis of correlation function (5.6) is quite different.

The behavior of the temporal correlation function at the condition $l_t \ll \lambda$, i.e., $t \ll \tau$, has been studied in detail [1-9]. In the $l_t \gg l$ region field correlations vanish entirely due to large values of t. We thus consider the intermediate interval $\lambda \ll l_t \ll l$, determining the temporal range $\tau \ll t \ll (D_sl^{-2})^{-1}$.

Calculating integrals (5.6), we use the following formula for a multiple Gaussian integral:

$$\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}dq_1\cdots dq_n\,\exp[-D_st(q_1^2+\cdots+q_{n+1}^2)] =\frac{1}{\sqrt{n+1}}\left(\frac{\pi}{D_st}\right)^{n/2} \tag{5.7}$$

at $\Sigma_{i=1}{}^{n+1}q_i=0$. An integral over q_l in Eq. (5.6) is mainly contributed from integration areas $|q_{lx}|$, $|q_{ly}| \leq (D_st)^{-1}$ due to the Gaussian exponential decay. Using the smallness of variables q_{lx} and q_{ly} as compared to k', one can significantly simplify denominators under symbol of product in Eq. (5.6). Taking into account that vectors $\mathbf{q}$ and $\mathbf{k}'$ are directed along axis z, and neglecting the terms of second order in q_{lx} and q_{ly}, one obtains

$$(\mathbf{p}_l+\mathbf{q})^2-k^2\approx\left(k'+q_z-\sum_{m=1}^{l}q_{lz}\right)^2-k^2. \tag{5.8}$$

Thus transversal variables q_{lx} and q_{ly} are contained only in the Gaussian exponential, and integrals over them can be taken by means of Eq. (5.7). The integral over q_{lz} is evaluated quite differently. At $q_{lz} \leq \sigma$ one has $D_stq_{lz}^2 \leq D_st\sigma^2 \ll 1$. Therefore one can neglect the exponential decay $\exp(-D_st\Sigma_{l=1}{}^{n+1}q_{lz}^2)\approx 1$, and integrate by means of the residue theorem

$$\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}dq_{1z}\cdots dq_{nz}\prod_{l=1}^{n}\frac{1}{[q_{1z}+\cdots+q_{lz}-q+(i\sigma/2)][q_{1z}+\cdots+q_{lz}-q-(i\sigma/2)]}=\left(-\frac{2\pi i}{q-i\sigma}\right)^n. \tag{5.9}$$

Also calculating the integral over q_z which contains a pole of $(n+2)$ order,

$$(-2\pi i)^n\int\frac{dq_z}{(q_z-i\sigma)^{n+2}}\exp(iqL)=-\frac{(2\pi L)^{n+1}}{(n+1)!}e^{-\sigma L},$$

we obtain the transmission correlation function as a series in scattering orders

$$C_E^{(T)}(t)=BLe^{-\sigma L}\sum_{n=0}^{\infty}\frac{(\sigma L)^n}{n+1}\left(\frac{\tau}{4t}\right)^n\frac{1}{(n+1)!}. \tag{5.10}$$

As seen from Eq. (5.10) the number of scattering orders to be accounted for increases with the slab thickness, and decreases with time.

One can formally sum series (5.10) presenting the correlation function in a closed form,

$$C_E^{(T)}(t)=\frac{B}{\sigma}\left(\frac{4t}{\tau}\right)e^{-\sigma L}\int_0^{\kappa}\frac{dx}{x}(e^x-1), \tag{5.11}$$

where $\kappa=\sigma L\tau/(4t)$.

The temporal correlation function $C_E^{(T)}(t)$ can readily be calculated from Eqs. (5.10) or (5.11). For sufficiently thick slab, $\sigma L=10$, the double-scattering contribution is calculated to be about half of the total function at $t/\tau=1$. The triple-scattering contribution approximates therewith near 25%, and scattering orders higher than the fifth practically do not change the correlation function within the considered time interval.

The relative weight of higher scattering orders decreases with decreasing slab thickness. For $\sigma L=2$ the double scattering contributes near 90% of the total sum at $t/\tau=1$.

VI. DOUBLE-SCATTERING CONTRIBUTION TO THE TRANSMISSION TEMPORAL CORRELATION FUNCTION

Since for sufficiently large times $t/\tau>1$, and slab thicknesses of the order of the extinction length, the double scattering closely approximates the transmission correlation function, we calculate its contribution $C_{E,2}^{(T)}(t)$ to the correlation function valid for arbitrary values of t/τ and σL.

This contribution is described by the term with $n=1$ in Eq. (2.7). Approximating propagators $T^*(\mathbf{r}_2'-\mathbf{r}_1')$ in the range $|\mathbf{r}_2-\mathbf{r}_1| \gg |\mathbf{r}_1-\mathbf{r}_1'|$, $|\mathbf{r}_2-\mathbf{r}_2'|$ as

$$T^*(\mathbf{r}_2'-\mathbf{r}_1')\approx T^*(\mathbf{r}_2-\mathbf{r}_1)\exp[i\mathbf{k}_{21}(\mathbf{r}_2'-\mathbf{r}_2-\mathbf{r}_1'+\mathbf{r}_1)],$$

we obtain

$$C_{E,2}^{(T)}(t)=B\int\int d\mathbf{r}_1 d\mathbf{r}_2 \frac{e^{-\sigma|\mathbf{r}_1-\mathbf{r}_2|}}{|\mathbf{r}_1-\mathbf{r}_2|^2} e^{-\sigma z_1-\sigma(L-z_2)} \times\exp\{-D_s t[(\mathbf{k}_i'+\mathbf{k}_{12})^2+(\mathbf{k}_s'+\mathbf{k}_{12})^2]\}, \tag{6.1}$$

where $\mathbf{k}_{12}=k'(\mathbf{r}_1-\mathbf{r}_2)/|\mathbf{r}_1-\mathbf{r}_2|$ is the wave vector of radiation propagating between two scattering points. Spatial integrals are taken over an infinitely wide slab with thickness L.

Introducing the relative coordinates $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$, $\mathbf{r}=(x,y,z)$, and transforming the spatial integral to the spherical coordinate frame, we have

$$C_{E,2}^{(T)}(t)=2\pi B\exp(-\sigma L)\int_{-L/2}^{L/2}dz_1\left(\int_0^1 du\int_0^{(L-2z_1)/(2u)}dr + \int_{-1}^0 du\int_0^{-(L+2z_1)/(2u)}dr\right) \times\exp\{-[\sigma r+4D_s tk^2](1-u)\}. \tag{6.2}$$

Taking integrals with respect to r and z, we obtain the transmission correlation function within the double-scattering approximation in the form

$$C_{E,2}^{(T)}(t)=\frac{B}{2}L\ \exp(-\sigma L-4D_s tk^2)\left\{\int_0^1 du\frac{\exp(4D_s tk^2u)}{1-u} \times\left\{1-\frac{u}{\sigma L(1-u)}\left[1-\exp\left(-\frac{\sigma(1-u)L}{u}\right)\right]\right\} + \int_{-1}^0 du\frac{\exp(4D_s tk^2u)}{1-u}\left\{1-\frac{u}{\sigma L(1-u)} \times\left[\exp\left(\frac{\sigma(1-u)L}{u}\right)-1\right]\right\}\right\}. \tag{6.3}$$

Equation (6.3) describes the double-scattering transmission for a wide beam $A^{1/2}/L\gg 1$, incident normally to the slab. For a scattered wave deviated from the normal direction, Eq. (6.3) remains valid to terms of order of θ^2.

The double-scattering term $C_{E,2}^{(T)}(t)$ calculated with Eq. (6.3) is shown in Fig. 5 as a function of time, for various slab thicknesses. It appears to coincide with the temporal correlation function (5.10) contributed by all scattering orders for $t/\tau\geqslant 3$. The decay of $C_{E,2}^{(T)}(t)$ becomes more sharp with decreasing thickness. Comparing results of the calculation with Eqs. (5.10) and (6.3) one sees that the temporal correlation function can be adequately approximated by the double scattering for $\sigma L\leqslant 2$.

VII. SUMMARY

We calculated the temporal correlation function of multiply scattered light for backscattering and transmission. We developed a method for calculating the correlation function in terms of t/τ. Applying it to the backscattering and taking into consideration terms up to the $(t/\tau)^2$ order, we obtained results that quantitatively agree with the known experimental

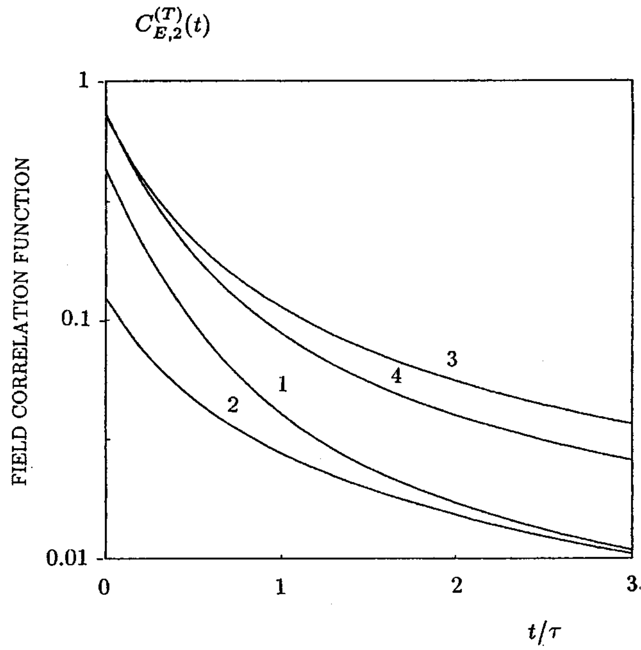

FIG. 5. Transmission temporal correlation function of field within the double-scattering approximation: (1) $L/\lambda=0.5$; (2) $L/\lambda=5$; (3) $L/\lambda=2$; (4) $L/\lambda=1$.

data. We ascribed the deviation of the linear dependence of the correlation function on $\sqrt{t/\tau}$ at small times to the boundedness of scattering orders because of the finite slab thickness, pointing out an illustrative resemblance to experimental plots. Note that, comparing the calculated results with measurement data, we substituted l^* instead of l, considering it a reasonable fit [22] for results obtained within the pointlike scatterer approximation. Considerable attention is being focused currently on the problem of scatterer finiteness [12,23–25].

We calculated the temporal correlation functions for polarized and depolarized scattered light components. The temporal slope for the polarized component was shown to be quite sensitive to the scattering angle, whereas the slope for the depolarized one practically does not vary with the angle.

Within the factorization approximation the intensities of scattered light with the same polarizations only give rise to nonzero correlations, since $\langle\delta E_x(0)\delta E_y^*(t)\rangle=0$ from the symmetry consideration. Going beyond the framework of this approximation, the connected eight vertex diagrams should be considered, involving a nonzero contribution to the correlation of intensities with different scattered light polarizations, $\langle\delta E_x(0)\delta E_x^*(0)\delta E_y(t)\delta E_y^*(t)\rangle\neq 0$. Graphically the transition to connected diagrams is performed by means of the Hikami vertex [26], and corresponds formally to accounting for the higher-order terms in parameter λ/l. The factorization approximation describes the main, short-range part of the correlation function, whereas the connected diagram contributions reveal themselves as long-range parts weaker in magnitude [27]. Generally the long-range terms arising from the λ/l higher-order contributions become more prominent for larger values of t and can obscure the corrections of higher order in t/τ to the short-range term found here. This makes the study of this intensity correlation function with different light polarizations more important, for it does not contain the dominant short-range term.

A method for evaluating transmission correlation functions has been developed for $t/\tau > 1$ based on the asymptotics of multiple Gaussian integrals. For smaller time a diffusion theory has been found [28] by means of computer simulations to be accurate within 1% for $L > 5l^*$. The double-scattering term has been shown to describe adequately the correlation function for thinner slabs.

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